Menu Costs at Work: Restaurant Prices and the Introduction of the Euro*

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Abstract

Restaurant prices in the euro area increased dramatically after the introduction of the euro. We show that this increase can be explained by a common menu cost model, extended to include a state-dependent decision of firms on when to adopt the new currency. Two mechanisms drive this result. First, firms concentrate otherwise staggered price increases around the changeover. Second, before the adoption of the euro, prices do not reflect the marginal cost increases expected to occur after the changeover. This "horizon effect" disappears as soon as the new currency is adopted, causing a jump in the optimal price.

Keywords: menu costs, sticky prices, inflation, euro.

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I. Introduction

Before euro coins and bills started circulating in January 2002, many feared that the changeover could result in excessive price increases. In retrospect, this alarm appears largely misplaced. According to the European Commission, the increase in the all-item Harmonized Index of Consumer Prices attributable to the introduction of the euro was only between 0.12 percent and 0.29 percent. However, prices in particular sectors—most notably restaurants and cafes—increased significantly in the period immediately following the changeover.

In January 2002 alone, prices in the euro area restaurant sector increased by 1.3 percent. In comparison, the all-item monthly inflation rate was 0.16 percent. In most countries, restaurant inflation was much higher than its historical average, with peaks of 3.4 percent in the Netherlands and 2.1 percent in Germany. These price increases are not only striking when compared with the historical record; they also contrast sharply with those observed in the European Union (EU) countries that did not adopt the euro: Denmark (0.1 percent), Sweden (0.2 percent), and the United Kingdom (0.1 percent). This discrepancy between the two groups of countries clearly points to the introduction of the new currency as the most likely cause of the jump in prices.

What could explain such a remarkable effect from a change in currency denomination? One possibility is that the introduction of the euro acted as a coordination device, allowing restaurants to collectively raise their prices, as argued by Adriani, Marini and Scaramozzino [2003]. However, this story does not explain why it was the changeover, rather than any other event, that triggered the switch to a high-price equilibrium. Moreover, if the changeover was indeed a suitable focal point for expectations, it is not clear why price increases were not more widespread across sectors. Similar considerations apply to a second class of explanations, based on bounded rationality, as in Gaiotti and Lippi [2005] and Mastrobuoni and Dziuda [2005], in which consumers face some friction in translating prices from the old currency to the new. Finally, a third theory emphasizes the role of rounding to arrive at attractive prices. However, rounding alone cannot account for the systematic bias toward higher prices observed in the restaurant sector.

We offer an alternative explanation for the unusual behavior of restaurant prices documented above—the existence of menu costs. Central to our analysis is a pricing model for a small sector with monopolistic competition and costly price adjustment, modified to take into account the introduction of a new currency. In this model, as in Dotsey, King, and Wolman [1999], price setters face a stochastic cost of changing prices—a menu cost. In addition, they must pay a fixed cost for switching to the new currency. They also incur losses for switching "early" (in our case, before January 2002) or "late." These costs capture the fact that euro cash payments were not possible before January 2002 and that domestic currencies were quickly retired from circulation after the changeover.[1]

Our key assumption is that switching to the euro involves choosing new optimal prices. In particular, we posit that the costs incurred for the adoption of the euro already incorporate the costs normally faced when changing prices. In other words, firms that adopt the euro can also adjust their prices for free.

We study the response of restaurant prices to the announcement of the introduction of the euro in four variants of our pricing model, which correspond to some popular specifications in the literature. In particular, we consider: *i)* Calvo's [1983] time dependent model, *ii)* a hybrid of Calvo [1983] and Taylor [1980], and *iii)* the state dependent model of Dotsey, King, and Wolman [1999]. In these three cases, we assume for simplicity that all firms must adopt the euro in January 2002. To our surprise, we find that this assumption implies a much larger price jump than that observed in the data.

For this reason, we introduce a version of this model with an endogenous adoption margin. In this specification, we calibrate the cost parameters that affect the endogenous adoption decision to match the pace of adoption of the euro among small and medium enterprises (SMEs) in the euro area. For these particular parameter values, the model generates a jump in restaurant prices that is of the same order of magnitude as that in the data.

Two main mechanisms generate this jump. The first concentrates an unusually high fraction of otherwise staggered price changes at the time of the currency switch. The result is a price jump, followed by a protracted period of lower than average inflation as the distribution of price vintages "churns" back to its steady state. We refer to this as "distributional churning." The second mechanism, which we call the "horizon effect," depends on the fact that cost increases expected to occur after the changeover are not incorporated in the prices that firms set before the changeover.

[1] The euro became the official currency of the European Monetary Union in January 1999, when the exchange rates of the participating countries were irrevocably fixed. However, euro coins and bills started circulating on January 1, 2002. Therefore, pricing in euros was impractical before this date, at least for the purpose of cash payments. Moreover, domestic currencies quickly disappeared from circulation after the changeover. The euro became the sole legal tender in the Union on March 1, 2002. Therefore, pricing in domestic currency became quite costly almost immediately after January 1, 2002 and simply impossible at the end of February 2002.

The prices set at the time of the switch, however, do include those costs. As a consequence, in the periods leading up to the changeover, firms have an incentive to postpone price increases and to increase prices by less if they do. The result is a decline in inflation and a backlog of overdue price adjustments. This backlog is quickly eliminated as firms set prices in the new currency, thus contributing to the price jump.

This paper shows that the behavior of restaurant prices in the period surrounding the introduction of the euro is not puzzling from the perspective of economic theory. In fact, a menu cost model with a reasonable amount of price stickiness produces at least as much inflation in January 2002 as that seen in the data. In light of this model, the real puzzle is why price increases were not more widespread. Addressing this question is important because it could provide some insight into the empirical plausibility of sticky price models. We propose three tentative answers.

The first is simply that the kind of price rigidity captured by menu cost models is only relevant for a handful of sectors. This answer is consistent with the observation that the sectors with significant price jumps around the changeover are among those with the highest measured price stickiness, both in the Unites States [Bils and Klenow 2004] and in the euro area [Dhyne, Álvarez, Le Bihan, Veronese, Dias, Hoffman, Jonker, Lünnemann, Rumler, and Vilmunen 2004].

The second answer is that the switch to euro prices was not generally accompanied by a reoptimization, as we assume here. Some firms might have switched to the euro without reoptimizing because their "managerial" and "customer" costs of changing prices are significantly higher than the "physical" costs of updating their price lists, as found by Zbaracki, Ritson, Levy, Dutta and Bergen [2004] in their case study of a large industrial manufacturer. Indeed, adopting a new currency does require printing new menus, but it does not necessarily require reevaluating the prices in them. From this perspective, our model should be interpreted as one of purely "physical" menu costs. The paper's findings suggest that this is a reasonable first approximation for the restaurant sector.

Finally, restaurants that adopted the euro either early or late probably faced substantial transaction costs, since payments in the European restaurant sector are made predominantly in cash. Thus, restaurants might have been encouraged to concentrate their price increases around January 2002 more so than firms in less cash-intensive sectors. Unfortunately, we are not aware of any direct evidence on the pace of adoption across sectors that might corroborate this conjecture.

The structure of the paper is as follows. In section II, we present some evidence on the anomalous price increases in the euro area restaurant sector around January 2002. In section III, we introduce our theoretical framework, a partial equilibrium model of a small sector with monopolistic competition and sticky prices. The partial equilibrium setup is particularly appropriate for our purposes, since the introduction of the euro had negligible effects on the general price level but a big effect on restaurant prices. In section IV we describe the main mechanisms that determine the equilibrium inflation rate in the small sector, "distributional churning," and the "horizon effect." In section V, we show that, for realistically calibrated parameter values, the model with endogenous adoption of the euro generates a spike in inflation that is remarkably close to that observed in the data. Section VI concludes. Mathematical details are relegated to an external appendix, which is available upon request.

II. European Inflation and the Euro

In this section, we present evidence on the behavior of inflation in the European Union around the changeover to the euro in January 2002. We do so in two steps.

First, we look at overall inflation for the EU12 in the so-called Harmonized Index of Consumer Prices, or HICP.[2] We find that European inflation, as measured by changes in this index, was not unusually high around the time of the changeover. The same is true in each of the twelve countries that adopted the new currency. Second, we show that the aggregate evidence hides some significant price increases at the sectorial level. These increases are particularly striking for restaurants and cafes, which are the focus of this paper.

II.A. *Overall Inflation*

Figure I shows the evolution of the EU12 HICP inflation rate from January 1994 to March 2004. The data are not seasonally adjusted, since a seasonal filter would smooth out the kind of temporary price changes that we are interested in. This explains the series' high volatility.[3] As is evident from the figure, nothing remarkable happened in or around January 2002. The inflation rate in that month was 0.2 percent. Inflation peaked in March 2002 at 0.6 percent, but very similar

[2] The EU12 countries are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. Eurostat's harmonized CPI for the euro zone is compiled using country weights based on each country's share of private final domestic consumption expenditures in the zone total.

[3] According to Lünneman and Mathä [2004], the increase in volatility that is evident since January 2001 is largely attributable to changes in the treatment of sales.

peaks recurred in each March since 2001.

Of course, the aggregation of the underlying national inflation rates into the euro area index might very well hide a significant degree of national heterogeneity in the effect of the changeover. However, this is not the case. In all EU12 countries, price movements in or around January 2002 do not stand out against the backdrop of the general volatility and pronounced seasonality of the series. The only possible exceptions are Germany and Spain. In December 2001, the German monthly inflation rate hit 1.1 percent—the highest level in the sample. In Spain, inflation climbed to 0.9 percent and 1.4 percent in March and April of 2002.

On the whole, our reading of this evidence does not support the idea that the changeover to the euro had an unusual impact on European consumer price inflation. Furthermore, the inflation experience in the twelve countries converting to the euro in January 2002 was not very different from that in Denmark, Sweden, and Britain, which opted out of the Monetary Union.

Several studies, such as Deutsche Bundesbank [2004] and Del Giovane, Lippi and Sabbatini [2005], have reached similar conclusions. In fact, the European Commission's official answer to the question: "Has the introduction of euro notes and coins caused prices to rise?" reads

> "Analyses by Eurostat, the European Commission's statistical service, indicate that the euro changeover led to some price increases in specific sectors, such as restaurants, cafes and hairdressers, but that the overall effect on prices in the euro area was limited. For the all items Harmonized Index of Consumer Prices, the price increase most likely falls within the range 0.12 percent to 0.29 percent."[4]

However, this answer suggests that the introduction of the euro led to significant price increases in some sectors, like restaurants and cafes, to which we now turn our attention.

II.B. *Restaurant and Cafe Inflation*

Price increases in January 2002 and, in some countries, the subsequent few months, were exceptionally high among restaurants and cafes all across the euro area. Figure II shows the evolution of the monthly inflation rate in the EU12 HICP for this sector from January 1995 through March 2004. The average inflation rate over the period was 0.2 percent. In January 2002, inflation was

[4] See http://europa.eu.int/comm/economy_finance/euro/faqs/faqs_16_en.htm

1.3 percent. Even in the three months following January 2002, restaurant and cafe prices rose at an average rate of more than twice the overall average. These inflation rates are hardly dramatic in absolute value, and very far from the "rampant price gouging" denounced by some European citizens and observers [Arnold 2004]. However, these rates certainly stand out against the historical norm and represent a challenge to the standard economic prior that changes in currency denomination should have no effect on relative prices.

Moreover, similar price jumps are evident in all the euro area countries. In some of them, the increases were concentrated in January 2002. Finland, France, Germany, and the Netherlands, for example, experienced inflation rates between 1.5 percent and 3.5 percent. In other countries, like Italy and Spain, the price increases in January 2002 were more moderate, but persisted into the subsequent months.

The behavior of restaurant prices in the three European Union countries that did not adopt the common currency—Britain, Denmark and Sweden—further corroborates the hypothesis that the price increases documented above are connected to the changeover. None of these three countries, in fact, experienced more than average price increases in the first few months of 2002.

Table I summarizes this striking anomaly. It reports the inflation rate in restaurants and cafes in January 2002, denoted by $\pi_{Jan2002}$, the average inflation rate over the period 1995 to 2003, and its standard deviation. Among the EU12 countries, with the exception of Greece, inflation in January 2002 exceeded its historical average by between 0.3 percent and 3.1 percent. In the three non adopting countries, this difference is zero or negative. The column headed $\left(\pi_{Jan2002} - \overline{\pi}\right)/s$ scales the difference between inflation in January 2002 and the average by the series' standard deviation. Again, for all EU12 countries except Greece, inflation in January was between 1.75 and 9 standard deviations away from the mean. This number for the EU12 as a whole is 7.

Of course, part of the anomaly might simply be caused by a seasonal effect, since our data is not seasonally adjusted. We control for this effect by computing the deviation of the average level of inflation in the month of January from the overall mean, $\left(\overline{\pi}_{Jan} - \overline{\pi}\right)/s$. This statistic is reported in the last column of Table I. The calculation shows that inflation in January does tend to be higher than average. However, its magnitude exceeds two standard deviations only in Spain. This result confirms that the January 2002 observation is clearly anomalous, even after taking into account January's seasonal characteristics. In the remainder of the paper, we will try to make sense of this anomaly in light of commonly used menu cost models.

Before proceeding, however, it is important to stress that anomalous price increases following the changeover, although most pronounced among restaurants, were not unique to this sector. For example, several of the national studies of micro consumer prices conducted by the ECB's Inflation Persistence Network (IPN) find evidence of an increase in the frequency of price changes for some items in or around January 2002.[5] More specifically, Deutsche Bundesbank [2004] documents a significant effect of the euro changeover on the prices of cinema tickets, dry cleaning, hairdressing, and restaurants in Germany. Interestingly, these and other related items, like a glass of beer in a cafe, correspond to seven of the seventeen stickiest prices in Germany [Hoffman and Kurz-Kim 2004], and to seven of the eleven stickiest prices in the euro area. In fact, these seven prices change with a monthly frequency between three and five percent, as compared to an average frequency of fifteen percent for the whole sample. Although not overwhelming, this evidence suggests a connection between price stickiness and the inflationary effect of the euro. We explore this connection in the remainder of the paper.

III. The Introduction of the Euro in Sticky Price Models

This section describes how we embed the introduction of a new currency in an otherwise standard sticky price model.

Our starting point is a model with stochastic menu costs, along the lines of Dotsey, King and Wolman [1999], thereafter DKW. We adopt this framework because it encompasses several familiar sticky price models, given appropriate assumptions on the distribution of menu costs. In particular, we will consider: (*i*) a Calvo [1983] model; (*ii*) a hybrid of the models of Calvo [1983] and Taylor [1980], also studied by Klenow and Kryvtsov [2005]; and (*iii*) the state-dependent model of DKW. In these three models, the timing of the adoption of the new currency is exogenous. In addition, we introduce an augmented version of DKW—model (*iv*), in which firms decide endogenously when to adopt the new currency.

Our main goal is to compare the ability of these four models to generate a spike in inflation following the changeover. We model this event as follows. Suppose that the economy is on a balanced growth path, with a positive rate of inflation. At time 0, the government announces the introduction of a new currency, the euro, to occur at time $T > 0$. For firms, adopting the euro

[5] See the overview paper by Dhyne, Álvarez, Le Bihan, Veronese, Dias, Hoffman, Jonker, Lünnemann, Rumler, and Vilmunen [2004] and the references therein.

means changing the denomination of their prices. In general, this adjustment need not imply a change in "real" prices. However, we assume that the costs of adopting the new currency include the ordinary menu costs. Hence, in equilibrium, firms that switch to the euro will also choose new optimal prices. This assumption is crucial for our results. It can be rationalized in one of two ways.

A first possibility is to interpret menu costs narrowly, as the purely physical costs of replacing the menus. In this case, choosing new prices at the time of the switch can be done at zero marginal cost, since menus must be reprinted anyway. This might be a reasonable first approximation for restaurants.

A second possibility is to recognize that "translating" a restaurant menu from the old domestic currency to euros might require a "thinking" effort very similar to that of choosing entirely new prices. At least, restaurants had to reevaluate their euro prices to make them "attractive" in the new currency.[6] This suggests that the marginal cost of choosing fully optimal prices, conditional on switching to the euro, was probably low. Our assumption implies that this cost was in fact exactly zero.

Even though we consider a set of models that have been frequently analyzed in the literature, their application here is nonstandard. The introduction of the euro, and the price reoptimization that this brings about, represent an unusual shock. In particular, this shock causes the distribution of firms over the existing price vintages to collapse into a spike at the new optimal price. This collapse is then followed by a readjustment of the distribution toward its steady state. The nature of the experiment and the implied shift in the distribution are such that we cannot rely on log-linear approximations to solve the model. Therefore, we apply an extended path algorithm that allows us to solve exactly for the equilibrium price index of the restaurant sector, taking as given the evolution of the rest of the economy. Thus, our analysis brings to the forefront some unique nonlinear implications of the models, which are lost in the local analysis customary in the literature.

III.A. *The Model*

Our model describes the partial equilibrium of a small sector in a large economy. Demand for the sector's goods, as well as factor prices and trend inflation, depend on aggregate conditions. These conditions are taken as exogenous and assumed to be characterized by a balanced growth

[6] See Aucremanne and Cornille [2001] on the incidence of "attractive" prices and their role in the changeover.

path, with real growth g and a positive inflation rate π. Formally,

$$Y_t = (1+g)^t y,\ P_t = (1+\pi)^t p,\ W_t = [(1+g)(1+\pi)]^t w \text{ and } r_t = r, \tag{1}$$

where Y_t is output, P_t the aggregate price level, W_t the nominal wage, and r_t the nominal interest rate.

The model's small sector, denoted by i, is the theoretical counterpart of the restaurant sector in the data. It faces a constant elasticity demand function of the form

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} Y_t, \tag{2}$$

where P_{it} is an appropriately defined price index, discussed in more detail below. The sector is populated by a continuum of firms. They supply a differentiated product, indexed by $k \in [0,1]$, with demand

$$Y_{ikt} = \left(\frac{P_{ikt}}{P_{it}}\right)^{-\varepsilon} Y_{it} = \left(\frac{P_{ikt}}{P_{it}}\right)^{-\varepsilon} \left(\frac{P_{it}}{P_t}\right)^{-\eta} Y_t. \tag{3}$$

The parameter $\varepsilon > 1$ is the elasticity of substitution in the CES preferences that generate these demand functions. As a consequence, the price index for the sector is

$$P_{it} = \left[\int_0^1 \left(\frac{1}{P_{ikt}}\right)^{\varepsilon-1} dk\right]^{\frac{1}{1-\varepsilon}}. \tag{4}$$

This index is a key object in the analysis. It is the model's equivalent of the EU12 HICP for restaurants and cafes constructed by Eurostat. Our goal is to reproduce in the model the spike in this index observed in January 2002.

As in DKW, firms face a stochastic cost of adjusting their prices, denoted by ξ. Its realization is drawn independently by each firm in each period from a distribution with c.d.f.

$$G_t(\xi) = \begin{cases} 0 & \text{for } \ \xi < 0 \\ \gamma_{1t} + \gamma_{2t}\tan(\gamma_{3t}\xi + \gamma_{4t}) & \text{for } \ 0 \le \xi < \overline{\xi}_t \\ 1 & \text{for } \ \overline{\xi}_t \le \xi. \end{cases} \tag{5}$$

Note that this distribution has only three free parameters, since two of the five parameters are pinned down by the restrictions $G_t(0) = 0$ and $G_t\left(\overline{\xi}_t\right) = 1$.

Both Calvo's time-dependent formulation and the hybrid of Calvo [1983] and Taylor [1980] studied by Klenow and Kryvtsov [2005] can be thought of as limiting cases of this model. In fact, when $\overline{\xi}_t \to \infty$, and the tangent function becomes a step of height α, the menu cost is zero with probability α, and infinite with probability $1-\alpha$. This is the Calvo model. When instead $\overline{\xi}_t < \infty$, a fraction α of firms still draws a zero menu cost and always reoptimizes. In addition, the firms whose benefit from adjusting exceeds the maximum menu cost, $\overline{\xi}_t$, also reoptimize. This results in an endogenous upper bound on the distribution of price vintages, as in Taylor's [1980] staggered contracts model.

Firms in sector i operate a constant returns to scale technology with nominal marginal cost Ψ_t. The marginal cost is taken as given by each firm and grows along the balanced growth path according to

$$\Psi_t = (1+\pi)^t \psi. \tag{6}$$

Following most of the literature, we abstract from the kind of heterogeneity in marginal costs considered by Golosov and Lucas [2003]. They argue that the high volatility of idiosyncratic shocks is likely to offset the inertia in prices caused by menu costs. However, the fact that restaurant prices are among the stickiest in both the United States and Europe suggests that, in this sector, marginal cost shocks are small compared to menu costs.

Profits net of the menu cost can be written as

$$(P_{ikt} - \Psi_t)\, Y_{ikt} - W_t f_t \tag{7}$$

where f_t denotes the flow cost of "currency mismatch" in terms of labor. In particular, firms charging euro-denominated prices before time T must pay a cost of $f_t = f^E$ per period, while firms sticking to the old currency after time T spend an additional $f_t = f^D$ units of time on transactions. Moreover, we assume that firms must pay a one-time "switching" cost, denoted by c, as well as the menu cost ξ when they first start charging in the new currency.

Our first three models, in which we assume that all firms need to adjust at time T, correspond to the case $f^E = f^D = \infty$. Instead, in the model with endogenous adoption, these costs are finite and give rise to an endogenous margin for the adoption of the new currency. In this case, since the total adoption cost inclusive of the menu cost, $c+\xi$, is stochastic, firms have an incentive to postpone the switch in the hope of drawing a low menu cost. of course, firms also have an incentive to anticipate the switch, in the case their current draw is low. In this sense, the endogenous adoption decision is very similar to the regular decision to change prices, except that the cost of inaction also includes the revenue loss, $W_t f_t$.

Let $V_{0,t}^D$ denote the value of a firm, gross of the adjustment cost, if it adjusts its price at time t and decides to denominate its new price in the domestic currency. Similarly, let $V_{0,t}^E$ be the value of a firm that adjusts its price at time t and decides to charge its new price in euros. Let $V_{j,t}^D$ be the value at time t of a firm that set its domestic-currency-denominated price j periods ago and let $V_{j,t}^E$ be the value of a firm that set its euro-denominated price j periods ago. Let $\Pi_{j,t}^D$ denote the flow profits earned at time t by a firm that adjusted its domestic currency denominated price j periods ago. $\Pi_{j,t}^E$ is similarly defined for firms with a euro-denominated price.

The value function of a firm adjusting its price at time t in terms of the original currency can be written as follows

$$(8) \quad V_{0,t}^D = \max_{P_{D,t}^*} \left\{ \Pi_{0,t}^D + \frac{1}{1+r} E_t \max \left\{ V_{0,t+1}^D - W_{t+1}\xi_{t+1}, V_{0,t+1}^E - W_{t+1}\xi_{t+1} - W_{t+1}c, V_{1,t+1}^D \right\} \right\}.$$

That is, a firm that adjusts its price at time t and sets this adjusted price in terms of the domestic currency chooses its new price level, $P_{D,t}^*$, to maximize the sum of its current flow profits and the discounted continuation value.

This continuation value is the expected maximum net value of three options faced by the firm in the next period. The first option is to adjust its price in the next period and pay the menu cost, ξ. The second option is to adjust its price and change to the euro in the next period. The cost of this choice is the sum of the menu cost, ξ, and the euro adjustment cost, c. The third option is to continue into the next period with the price already set.

In this setting, the only source of heterogeneity across firms is the realized level of the adjustment cost, ξ_{kt}. However, this realization does not affect the firm's value ex ante, since the adjustment costs are independent over time and across firms. This implies that all firms that change their price and continue to denominate it in the domestic currency change it to the same level, $P_{D,t}^*$. For this reason, we dropped the firm index k in the equation above.

A firm that changes its euro-denominated price at time t has the value

$$(9) \qquad V_{0,t}^E = \max_{P_{E,t}^*} \left\{ \Pi_{0,t}^E + \frac{1}{1+r} E_t \max \left\{ V_{0,t+1}^E - W_{t+1}\xi_{t+1}, V_{1,t+1}^E \right\} \right\}.$$

It again chooses its new price, in this case $P_{E,t}^*$, to maximize the sum of its flow profits plus the discounted value of its expected continuation options.

The continuation value is the expected maximum of only two options. The first is to change the price in period $t+1$, thus incurring the menu cost, ξ. The second is to continue at the existing price. This firm faces one less option in the next period than the firm that is still charging its prices in the domestic currency because we assume that the switch to the euro is irreversible.

Finally, the value of a firm that does not adjust its price and still charges in terms of its domestic currency is

$$V_{j,t}^{D} = \Pi_{j,t}^{D} + \frac{1}{1+r} E_t \max\left\{V_{0,t+1}^{D} - W_{t+1}\xi_{t+1}, V_{0,t+1}^{E} - W_{t+1}\xi_{t+1} - W_{t+1}c, V_{j+1,t+1}^{D}\right\}, \tag{10}$$

while a firm that set its price in euros j periods ago has value at time t equal to

$$V_{j,t}^{E} = \Pi_{j,t}^{E} + \frac{1}{1+r} E_t \max\left\{V_{0,t+1}^{E} - W_{t+1}\xi_{t+1}, V_{j+1,t+1}^{E}\right\}. \tag{11}$$

As before, the option of charging in the old domestic currency does not appear in the continuation value because of the irreversibility of the switch to the euro.

IV. Equilibrium Inflation Dynamics

What happens to the path of inflation in the restaurant sector in response to the announcement of the introduction of the euro? In this section, we describe the two basic mechanisms that determine the transitional equilibrium path of inflation: (*i*) distributional churning and (*ii*) the horizon effect. We limit ourselves to the intuitive illustration of these concepts and the way that they influence the evolution of prices in equilibrium. Some of the analytical details can be found in the appendix.

The equilibrium dynamics for sector i after the announcement of the introduction of the euro are determined by three factors: (*i*) how many firms in each period update their prices, (*ii*) what fraction of firms chooses to change denomination, and (*iii*) how much these firms raise their prices.

These quantities reflect, in turn, the evolution of two classes of equilibrium objects. The first is the prices set by reoptimizing firms. As argued above, there are only two such prices: $P_{D,t}^{*}$, for all the firms that update their price at time t and choose the domestic currency; and $P_{E,t}^{*}$, for the firms that charge in euros. The second important equilibrium variable is the distribution of firms over the relevant state space. At any point in time, the two factors that distinguish firms and are relevant for their price setting behavior are (*i*) how long ago they changed their price, indexed by j; and (*ii*) in what denomination that price is charged, indexed by $S \in \{D, E\}$. We denote the fraction of firms in period t that changed their price j periods ago and that charge it in denomination $S \in \{D, E\}$ as $\omega_{j,t}^{S}$.

In equilibrium, the set of price vintages is countable. Hence, the equilibrium price level can be written as

$$P_{it} = \left[\sum_{S \in \{D,E\}} \sum_{j=0}^{\infty} \omega_{j,t}^{S} \left(\frac{1}{P_{S,t-j}^{*}} \right)^{\varepsilon - 1} \right]^{\frac{1}{1-\varepsilon}}. \tag{12}$$

From this expression we see that, given the evolution of the distribution $\left\{ \omega_{j,t}^{S} \right\}_{j}$, the path of restaurant prices is completely characterized by the sequence of optimal prices $\left\{ P_{S,t}^{*} \right\}_{t}$.

As in Calvo [1983] and DKW, firms set their price as a markup over a weighted average of current and future levels of marginal costs. More formally,

$$P_{S,t}^{*} = \frac{\varepsilon}{\varepsilon - 1} \sum_{j=0}^{\infty} \Omega_{j,t}^{S} \Psi_{t+j} = \frac{\varepsilon}{\varepsilon - 1} \Psi_{t} \sum_{j=0}^{\infty} \Omega_{j,t}^{S} \left(1 + \pi\right)^{j} \text{ for } S \in \{D, E\} \tag{13}$$

where $\varepsilon / \left(\varepsilon - 1\right) > 1$ is the gross markup and the weights $\left\{ \Omega_{j,t}^{S} \right\}_{j}$ sum to one. These weights depend on three things.

The first is the discount factor $\lambda = \left(1 + g\right)\left(1 + \pi\right) / \left(1 + r\right) < 1$. The lower the discount factor, the less the firm cares about future profits relative to current profits. This reduces the degree to which the firm takes into account future marginal costs in setting the current price.

The second is the probability of not having adjusted the price j periods after time t. The intuition is as follows. Under flexible prices, the firm always chooses its price to equal the markup times the current level of marginal cost. However, when it is not sure when it will adjust its prices again, the firm takes into account not only current marginal costs, but also the marginal cost levels that it is likely to face before adjusting its price again. The more likely a firm is to face a particular marginal cost level, the more weight this future cost gets in the price setting policy. When a firm adjusts its price at time t, the likelihood of facing a marginal cost level of Ψ_{t+j} before adjusting its price again is determined by the probability of not having adjusted the price again j periods after time t.

The final factor affecting the weights is the effect of the sector's price level on the demand for a firm's good. As long as a firm does not adjust its price, the demand for its good will drop relative to the sector's price level P_{it}. The effect of the level of P_{it} on the demand for firm k is given by the demand function

$$Y_{ikt} = P_{ikt}^{-\varepsilon} P_{it}^{\varepsilon-\eta} P_t^{\eta} Y_t. \tag{14}$$

Hence, whenever the within-sector price elasticity of demand, ε, is higher than the price elasticity of demand across sectors, η, an increase in P_{it} increases the marginal revenue of firm k. In that case, if all other firms charge a higher price, contributing to an increase in P_{it}, firm k will want to charge a higher price as well. Because an individual firm's best price-setting response is increasing in the prices set by the other firms, this is a strategic complementarity in the sense of Cooper and John [1988].

Two main mechanisms affect the path of P_{it} in sector i after the announcement of the introduction of the euro. We discuss them in turn.

Distributional churning. On the model's balanced growth path, a constant fraction of firms update their price in each period. Klenow and Kryvtsov [2005] find that this is approximately the case for most sectors in the United States

The introduction of the euro, however, causes an anticipated deviation from this pattern, inducing all firms to change their prices within a short time. This event results in a shift in the price distribution toward more recent prices. In particular, in models (*i*) through (*iii*), all the mass is at $j = 0$ at time T, when all the firms adopt the euro and update their prices. Because of the endogenous adoption decision, this shift is less pronounced in model (*iv*).

This churning of the distribution of prices has two effects on the level of inflation. First, it leads to higher inflation in or around time T. In that period, a disproportionate number of firms are raising their prices. Second, after the adoption of the euro, the distribution of prices has a relatively small variance. As a consequence, firms adjusting their prices in the subsequent periods will not raise them as much as they would in the steady state. This depresses inflation after time T. This effect is illustrated in the bottom panel of Figure III, which depicts the price adjustments in the steady state relative to those s^* periods after T. At time $T + s^*$, the distribution of firms according to how long ago they last adjusted prices is (approximately) truncated at s^*. As a result, average price increases are smaller for firms that change their prices.

Horizon effect. As we noted above, the weights of future marginal costs in the optimal price set in period t, $\left\{\Omega_{j,t}^S\right\}_j$, are decreasing in the probability of adjusting before $t+j$. The introduction of the euro implies that firms charging in the domestic currency will certainly change their price on or around time T. Consequently, prices set before the euro changeover do not take into account marginal cost increases expected to occur after its adoption.

This horizon effect leads to a decline in inflation before time T for two reasons. First, firms that adjust their prices need to increase them by less than they would in the steady state, since they know they will soon be adjusting again. Second, in the state dependent models, some firms choose to postpone their reoptimization for the same reason. But once a firm adopts the euro, its price adjustment horizon expands again. Therefore, its new price will reflect increases in marginal costs over a much longer horizon than before.

The horizon effect is illustrated in the top panel of Figure III. The thick line in the top half of the figure represents the path of future increases in marginal costs. Below are two weighting schemes. The first depicts the weights on future marginal costs applied by firms along the balanced growth path. The second is the truncated sequence of weights used by firms in models (i) through (iii) when setting their prices s^* periods before the introduction of the euro. In model (iv), these weights are not necessarily truncated, but they will decline rapidly around s^*, depending on the equilibrium speed of adoption.

As a result of the horizon effect, the prices set by euro adopters reflect future marginal cost increases that were not incorporated into the prices previously charged. This implies that these firms increase their prices at a much higher rate than their counterparts still charging in the old currency. Note that this phenomenon occurs only when nominal marginal costs are expected to increase in the future, as is the case when average inflation is positive.

The next section illustrates the contribution of these two mechanisms to the blip in inflation at the time of the introduction of the euro.

V. How Does the Model Compare to the Data?

In this section, we compare the price response of the model's small sector to the introduction of the euro, with the evolution of restaurant price inflation in the euro area around January 2002.

First, we calibrate the models' parameters. Those related to the balanced growth path are based on euro area averages for the period 1995 to 2003. We choose $r = 5.9$ percent, the long-term interest rate on government bonds; $\pi = 2.8$ percent, the inflation rate for restaurants and cafes;

and $g = 2.0$ percent, the growth rate of real GDP.

As for the demand parameters, we choose $\varepsilon = 11$, which corresponds to a 10 percent markup in the flexible price equilibrium. This is a common choice in the empirical literature on the New Keynesian Phillips curve [Galí, Gertler, and López-Salido 2001]. To pick η, note that the elasticity of substitution between goods is likely to be higher within rather than across sectors. This observation would imply $\varepsilon \geq \eta$. As a benchmark, we focus on the corner case $\eta = \varepsilon$. Our results are relatively insensitive to lower values of η.

Calibrating the parameters of the menu cost distribution is crucial for our results. Unfortunately, direct evidence on restaurants' menu costs is not available. However, what is important for our application is the stickiness implied by certain cost parameters, rather than their magnitude. For this reason, we calibrate the parameters of the menu cost distribution so that in the steady state the model replicates the degree of price stickiness observed for restaurants in the data.

The source of these data is a survey conducted by the Dutch Centraal Planbureau [2002], asking restaurant owners about the average annual number of regular price adjustments in their establishments. Thirteen percent of respondents reported that they adjust their prices on average less than once a year, 69 percent responded that they adjust once a year, while 18 percent reported adjusting their prices on average two to four times a year.[7] The main advantage of this survey is that it contains enough information to calibrate the three parameters of our model's cost distribution. Moreover, it is unique in its focus on the restaurant sector. The survey's main disadvantage is that it covers only one country. However, a comparison of the Dutch survey results for the overall economy, with those reported by Fabiani, Druant, Hernando, Kwapil, Landau, Loupias, Martins, Mathä, Sabbatini, Stahl, and Stokman [2004] for the euro area, suggests that the amount of price stickiness in the Netherlands is fairly representative of European pricing practices.

Note that the regular price adjustments for restaurants reported in the survey appear to be more frequent than the changes in the prices of individual restaurant items in the United States [Bils and Klenow 2004] and the EU [Dhyne, Álvarez, Le Bihan, Veronese, Dias, Hoffman, Jonker, Lünnemann, Rumler, and Vilmunen 2004]. This difference might be explained by the fact that printing a new menu, our interpretation of the "regular price adjustment" measured by the Planbureau, does not necessarily imply changing all the prices on the menu [MacDonald and Aaronson 2000]. In any

[7] In practice, our model implies that there is always a strictly positive fraction of firms that adjust their prices between five and twelve times a year. The calibration uses the approximation that 17 percent of firms adjust their prices two to four times a year and that the remaining one percent adjusts more than four times.

case, the relative price flexibility implied by our calibration would bias the results toward finding little effect of the changeover on prices. As we will see in the rest of this section, however, the main shortcoming of the first three models we consider is, in fact, the opposite. We would need even more price flexibility to fit the observed price jump.

V.A. *Calvo Model*

The first specification we consider is based on Calvo [1983], in which only a fraction α of restaurants can adjust its prices every period. We calibrate this parameter to match the 13 percent of Dutch restaurants that reset their prices on average less than once per year.

Figure IV depicts the monthly restaurant inflation rate in the euro area, along with the path implied by the Calvo model, for the four years from January 2000 through December 2003. The equilibrium path of inflation in the model shows a slight decline before January 2002. This is the result of the horizon effect. After January 2002, the model predicts an even bigger decline, a result of distributional churning, followed by a return to the steady state.

What is most striking about this picture, however, is the magnitude of the price jump predicted by the model in January 2002. The jump corresponds to a monthly inflation rate of 4 percent, compared to the 1.3 percent in the data. To be able to replicate the observed rate of inflation, we would need to assume that only 1.5 percent of restaurants adjust their prices less than once a year. Translated to a monthly frequency, this number implies an average price duration of three months. This compares with an average duration of price spells of thirteen to fifteen months in the euro area [Dhyne, Álvarez, Le Bihan, Veronese, Dias, Hoffman, Jonker, Lünnemann, Rumler, and Vilmunen, 2004] and of seven months in the United States [Bils and Klenow 2004]. Clearly, the Calvo model would need unrealistically high levels of price flexibility to match the magnitude of the January 2002 inflation flare.

The main explanation for this surprising result is the assumption that, at any point in time, a positive fraction of firms have not adjusted their prices for an arbitrarily long period. This implies that, when the euro is introduced in January 2002, some pizzas are still being sold at prices set in 1902! But the pizzerias that have not adjusted their prices for a century will change them at time T, along with all others, and increase them by a very high percentage. The smaller the α, the higher the fraction of this type of restaurant, and the more pronounced the inflation spike.

In traditional analyses of the Calvo model, this assumption is not very important. The reason is that they rely on a log-linear approximation around a steady state with no inflation, in which the distribution of firms over price vintages is degenerate. On the contrary, in our experiment the introduction of the euro represents an exogenous shock to the timing of price setting, which results in a reshuffling of the distribution of firms. In the ergodic state, only a fraction α of the pizzerias with 1902 prices resets them each period. Moreover, with zero average inflation, old prices need not be very far from the current optimal one. In our experiment, instead, all the pizzerias update at time T. In addition, a positive average inflation makes the 1902 prices a very small fraction of their current desired level, causing a very large jump at the time of adjustment. These two facts together are behind the inflation spike produced by the model.

V.B. *Calvo-Taylor Hybrid and DKW Model*

The presence of century-old prices in the Calvo model is clearly counterfactual. However, we can easily eliminate this undesirable feature of the model by considering a finite maximum menu cost, $\bar{\xi}$, which leads firms not to change their prices for only a finite number of periods. The result is a hybrid of the Calvo and Taylor models, our model *(ii)*.

Simulating this model requires the choice of two parameters: the probability of facing no menu costs, α, and the size of the maximum menu cost, denoted by $\bar{\xi}$. In line with the Dutch evidence from Centraal Planbureau [2002], we calibrate them so that, along the balanced growth path, 13 percent of restaurants change their prices less than once a year and 86 percent do so between one and four times a year. This calibration implies that the average menu cost incurred by firms resetting their price is only 0.03 percent of their monthly revenue.[8]

Figure V depicts the equilibrium path of inflation in the Calvo-Taylor hybrid model. As expected, truncating the distribution of firms over price vintages reduces the inflation spike in January 2002 by about half. Inflation drops from 4 percent in the Calvo model to 2.5 percent in the Calvo-Taylor hybrid. However, the inflation spike is still two times as high as that seen in the data.

An interesting feature of this model is that it produces inflationary "echoes" following the switch to the euro. For our calibration, these echoes have a fairly long period of twenty-three months and are not very pronounced. In Figure V, they are recognizable in the 0.4 percent price increase in

[8] The average cost incurred is very low because most of the adjusting firms in this model do not pay any menu cost. The maximum menu cost is 5 percent of revenue. The oldest price in the distribution is twenty-one months old.

November 2003. The problem with these echoes is that they would become more pronounced if we reduced the upper bound on the menu costs, $\overline{\xi}$. But this is what is needed to reduce the inflation blip implied by the model. Either way, the model clearly has counterfactual implications.

There are several ways to eliminate the echoes implied by the hybrid model. One possibility is to introduce some noise by adding a dimension to the state space. This is the approach followed by Golosov and Lucas [2003], who assume that firms face idiosyncratic marginal cost shocks. Here, we follow a simpler route, since the heterogeneity of marginal costs is not central to our analysis. In particular, we spread the probability mass of the menu cost distribution over the interval $\left(0, \overline{\xi}\right)$, according to the c.d.f. (5).

Simulation of this model requires calibrating three parameters. We pick them by matching the three observations about price adjustment frequencies in the Dutch restaurant sector reported in section V. The calibrated parameter values imply an average menu cost equal to 0.46 percent of monthly sales and a maximum price duration of twenty-five months. These values are well within the range found in the literature.

The results for this specification are depicted in Figure V. As anticipated, adding probability mass to $\left(0, \overline{\xi}\right)$ smooths out the echo effects. However, it does not significantly contribute to reducing the inflation spike implied by the model. The next step then is to allow firms to choose when to make the transition to the euro. This step is taken up in the next section.

V.C. *Endogenous Adoption of the Euro*

In the simulations above, we always assume that all firms adopted the euro in January 2002. However, a series of surveys conducted by Gallup Europe on behalf of the European Commission before and after the changeover suggests that this was not the case in practice. For example, the survey conducted in October 2001 found that 29 percent of small and medium enterprises (SMEs), i.e., those with less than 250 employees, already invoiced in euros or planned to do so before January 2002, while 62 percent claimed they would switch to the new currency in January 2002. The rest was planning to make the transition after January 2002. The numbers are almost identical for firms with less than nine employees.

If we take this evidence as representative of what happened in the restaurant sector, which in Europe is dominated by small enterprises, the simulations above clearly exaggerate the impact of the changeover on the incidence of price reoptimizations in January 2002. A more realistic model of the adoption process might therefore help reconcile the theory with the data. This extension results in an augmented version of DKW, our model (*iv*).

To implement this model, we must calibrate three additional parameters. They are the flow costs of anticipating or delaying the adoption before or after January 2002, f^E and f^D, and the fixed cost incurred when actually adopting the new currency, c. This is just a fixed "surcharge" on the usual menu cost, representing one-time expenses like updating the cash register. It is calibrated to 7.5 percent of average monthly revenue, the cost of euro adoption for Dutch restaurants reported by the Centraal Planbureau [2002].[9] As for the costs of currency mismatch, f^E and f^D, they are chosen to replicate the pace of euro adoption for SMEs reported by Gallup Europe [2001]. Note, however, that the absolute level of these costs is inconsequential for our results. What matters instead is the marginal cost of the switch, $c+\xi$, relative to the cost of not switching, which includes the currency "mismatch" and the lost profit from a suboptimal relative price.

The resulting equilibrium path of restaurant price inflation is plotted in Figure VI. At 1.6 percent, the simulated inflation blip is remarkably close to the 1.3 percent observed in the data. Because of the calibrated 29 percent of firms adopting in December 2001, the inflation rate in that month is also somewhat higher than the 0.26 percent in the data. On the other hand, distributional churning depresses the inflation rate in the first few months of 2002. In the data, however, inflation remains above average for several months, suggesting that the euro adoption process might have been slower in the restaurant sector than in the sample of SMEs.

Shifting more firms towards a late adoption would also contribute to a further reduction in the inflation spike predicted for January, bringing the model even more in line with the data. However, the scope for such a shift is limited, since national currencies ceased to be legal tender in the European Union at the end of February 2002. Therefore, the adoption process must have been completed by then. The model closely replicates this observation. In fact, it implies an adoption rate for February of 98.5 percent.

Figure VI also highlights another interesting implication of the model. Inflation is lower than average in the period leading to the changeover due to the horizon effect, and for a few months afterward as a result of distributional churning. This behavior is quite intuitive, since the changeover

[9] This number is computed by the Planbureau on the basis of a survey conducted by the Dutch National Bank, which estimated adoption costs for the Dutch restaurant sector of about 95 million euros, on annual revenues of 15 billion.

should have no effect on the price level in the long-run. Quantitatively, however, this effect is not very pronounced, and is therefore hard to detect in the noisy disaggregated data.

In conclusion, we find that a menu cost model with endogenous adoption closely replicates the response of restaurant prices to the introduction of the euro, especially with regard to the inflation flare of January 2002. The state-dependent choices of both the timing of price changes and the adoption of the new currency are an integral part of the model's ability to replicate the facts. The former insures that prices are never too far from their optimal level, thus limiting the amount of latent inflation stirred up by the changeover. The latter spreads this inflation over several periods, rather than concentrating it entirely in January 2002.

VI. Conclusion

The increase in restaurant prices in the euro area that coincided with the introduction of the euro in January 2002 was unprecedented. Countries like the Netherlands and Germany witnessed monthly price increases of the same magnitude as those usually experienced in a year.

We argued that these price increases, although extraordinary, were not surprising in light of economic theory. In a reasonably calibrated model with sticky prices and an endogenous decision to adopt the euro, a spike in inflation is the natural consequence of the repricing that accompanies the switch to the new currency. The key assumption in this respect is that posting menus in euros for the first time lead restaurants to choose new prices as well. This action has two effects on inflation.

First, a higher fraction of firms updates their prices at or around the time of the changeover, thus raising inflation. We refer to this effect as "distributional churning," since it represents a shock to the distribution of price vintages. Second, these firms also raise their prices at a higher rate than in the absence of the new currency. This is the "horizon effect." Since firms know that they will have the opportunity to adjust their prices when they switch to the euro, the prices they set before that time do not reflect the increases in marginal costs expected to occur after the adoption. This effect actually depresses inflation in the period leading to the changeover. As soon as firms adopt the new currency, however, their new prices will reflect the future cost increases, contributing to the jump in inflation.

In this paper, we highlighted data from only one European sector—restaurants and cafes—in which the inflationary effect of the euro was particularly pronounced. We think that this sector's response to the changeover was unusual for three main reasons. First, its prices are very sticky. Second, the costs normally incurred to update its prices were already part of the euro adoption costs. Third, restaurants concentrated their transition to the euro around January 2002 because of the cash-intensive nature of their business.

Finally, we also focused on one particular quantitative prediction of the model—that of a spike in inflation in January 2002. But the model has several more testable implications. For example, it establishes a tight relationship between the average duration of prices and the magnitude of the inflation spike. Moreover, it predicts a decline in inflation before and after the changeover but no long run effects on relative prices. Testing these implications across countries and goods is a promising avenue for future research.

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TABLE I:

EU15 restaurant and cafe inflation in January 2002

Country	$\pi_{Jan2002}$	mean ($\overline{\pi}$)	std.dev.(s)	$\left(\pi_{Jan2002} - \overline{\pi}\right)/s$	$\left(\overline{\pi}_{Jan} - \bar{\pi}\right)/s$
Austria	0.5	0.2	0.2	2.0	0.8
Belgium	0.5	0.2	0.2	1.7	-0.1
Finland	2.0	0.2	0.3	5.6	1.0
France	1.4	0.2	0.2	8.1	1.5
Germany	2.1	0.1	0.2	9.0	0.8
Greece	0.3	0.5	2.1	-0.1	0.1
Ireland	1.2	0.4	0.4	2.0	0.5
Italy	0.8	0.3	0.2	3.3	0.4
Luxembourg	1.3	0.2	0.2	4.7	1.1
Netherlands	3.4	0.3	0.4	7.4	1.8
Portugal	1.4	0.3	0.4	2.8	1.0
Spain	1.2	0.3	0.2	3.7	2.4
EU12	1.3	0.2	0.2	6.9	1.7
Denmark	0.1	0.2	0.2	-0.4	0.3
Sweden	0.2	0.2	0.3	0.0	-0.2
Britain	0.1	0.3	0.1	-1.3	-0.6

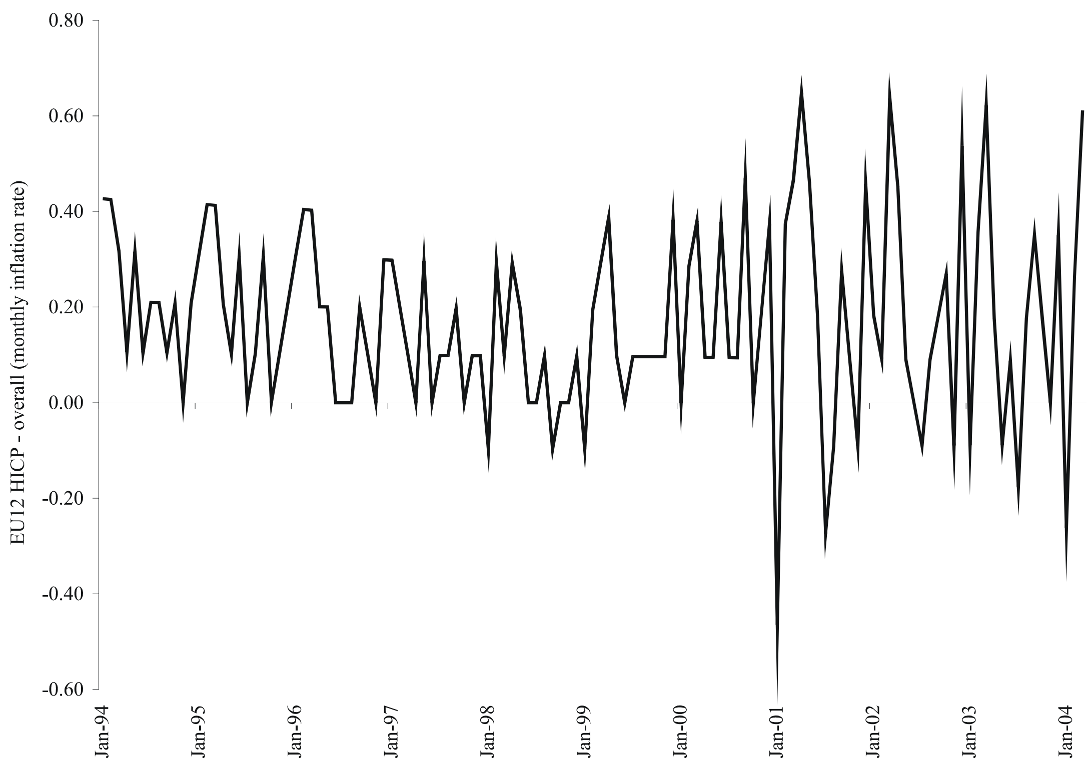

Figure I: EU12 overall inflation

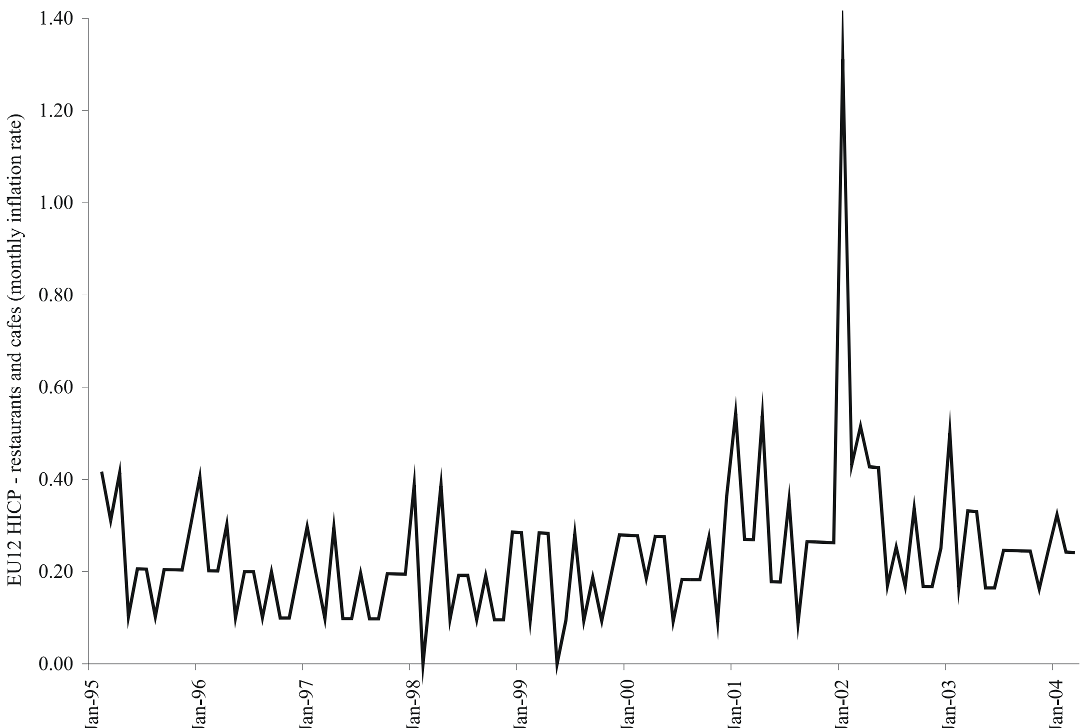

Figure II: EU12 restaurant and cafe inflation

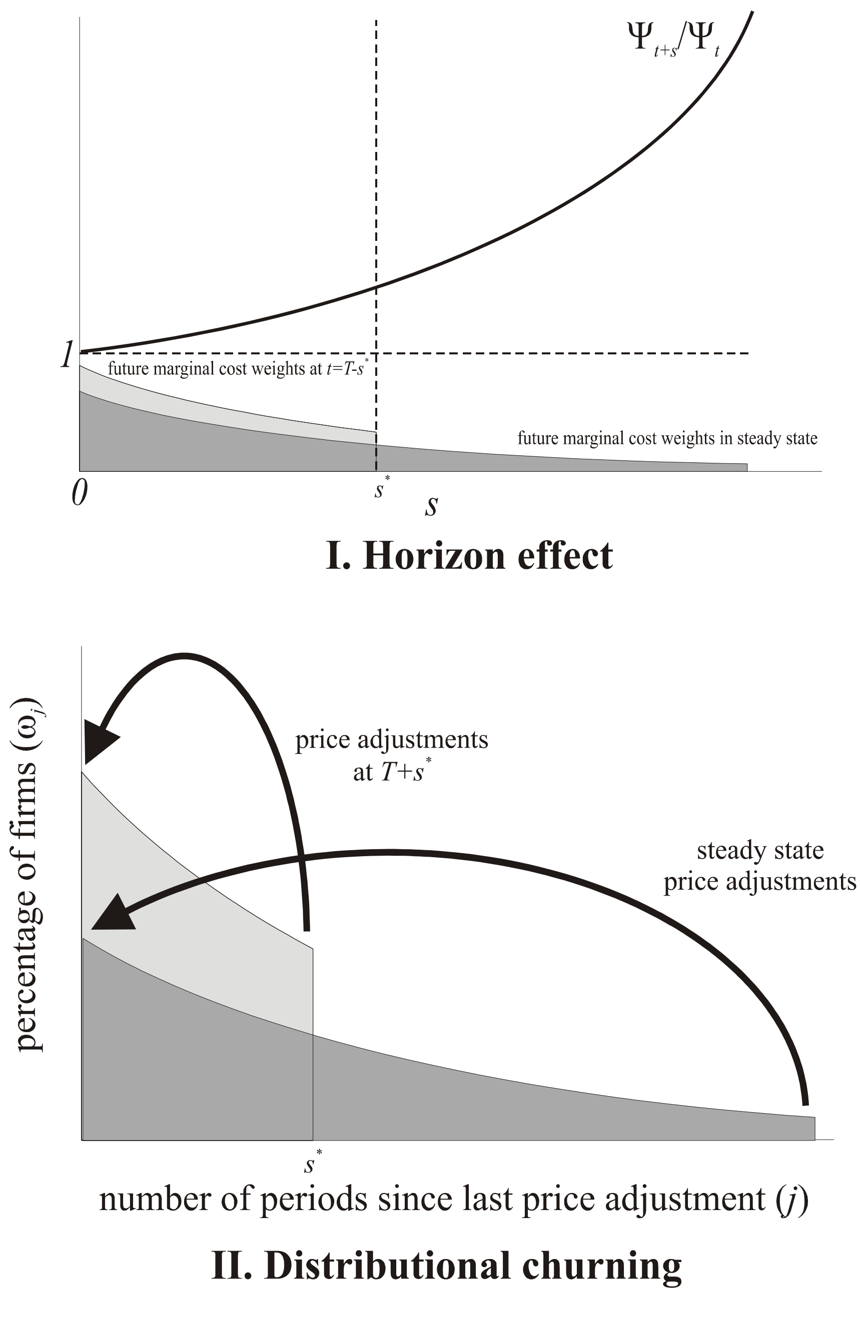

Figure III: The main mechanisms in the model's equilibrium inflation dynamics

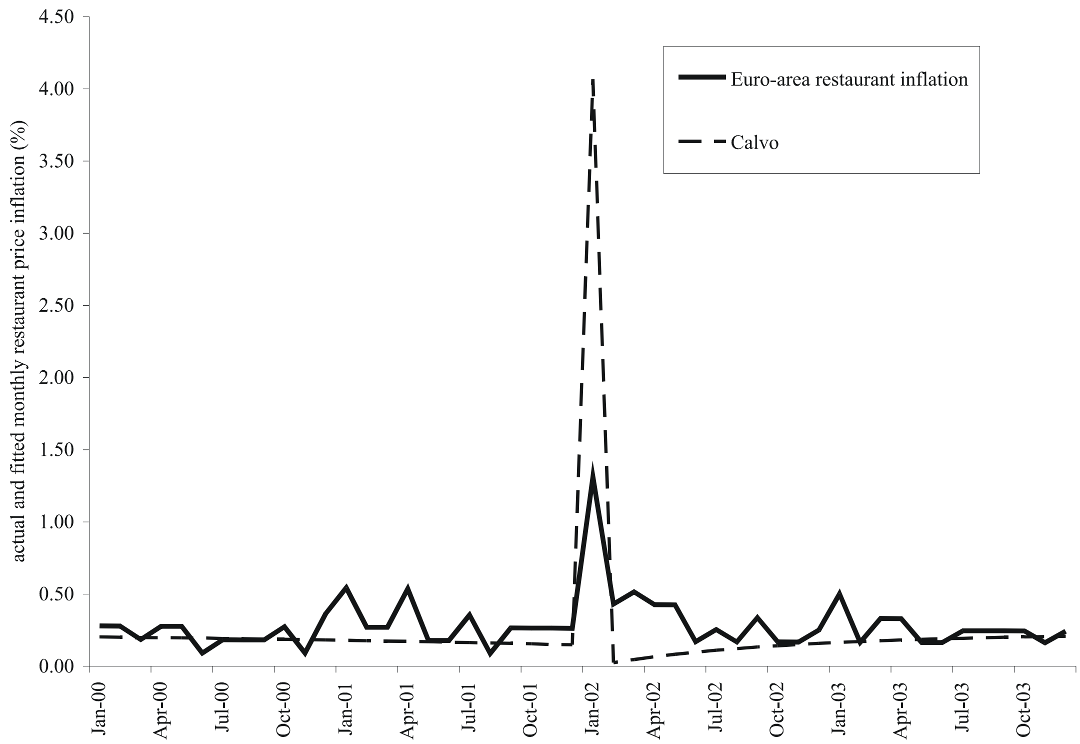

Figure IV: Calvo model

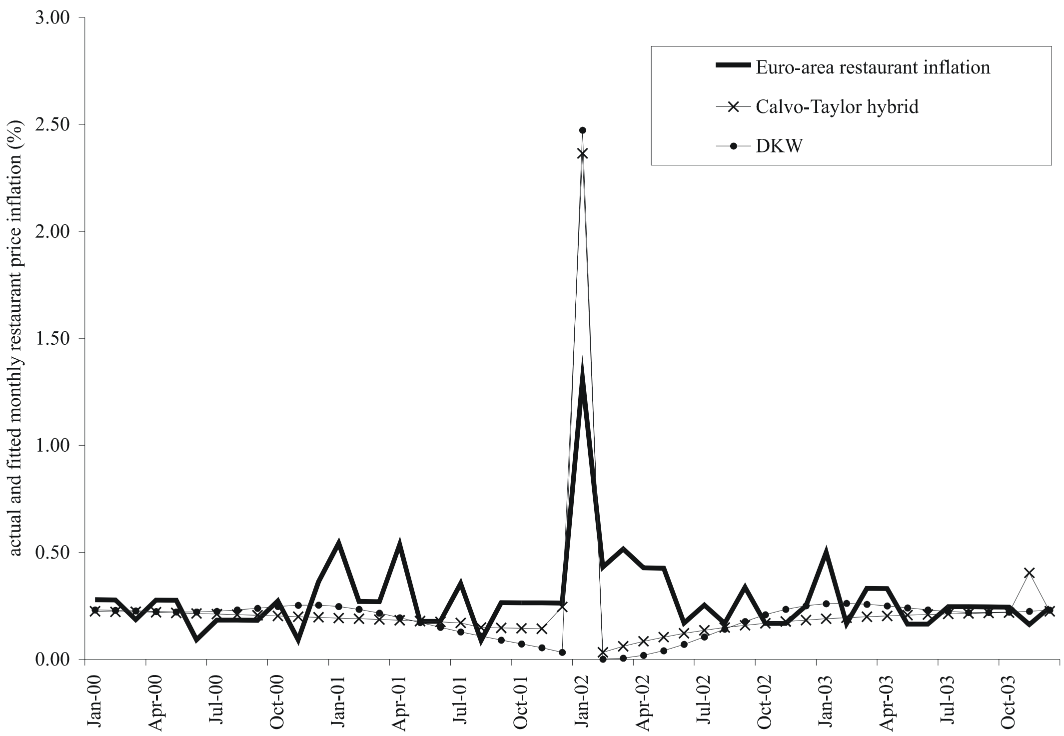

Figure V: Calvo-Taylor hybrid and DKW model

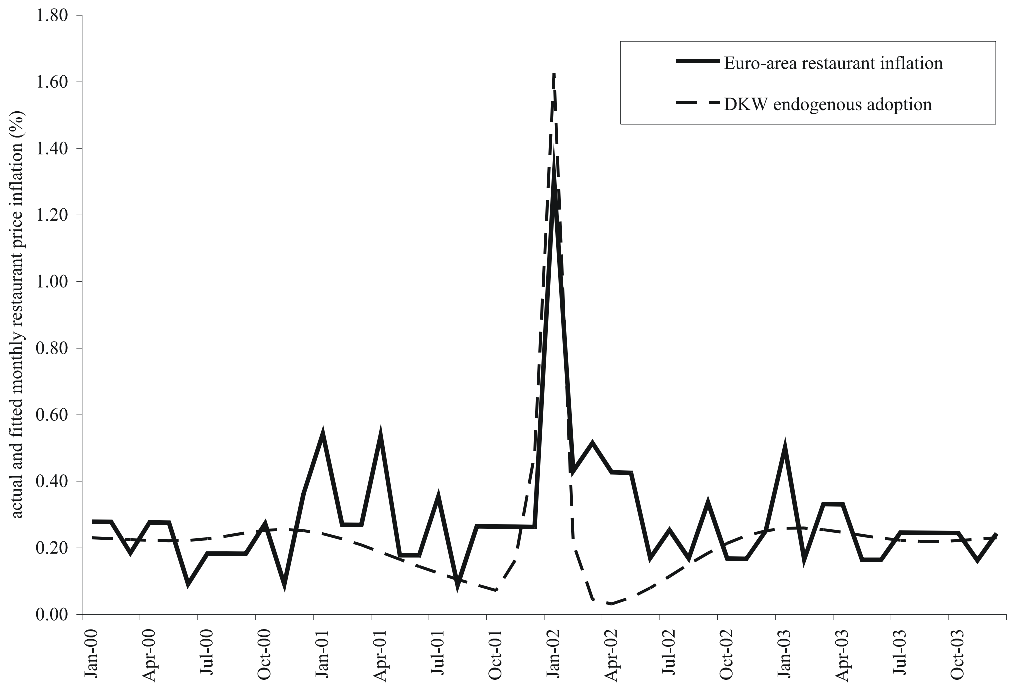

Figure VI: Augmented DKW model with endogenous adoption of the euro

Menu Costs at Work: Restaurant Prices and the Introduction of the Euro

Technical Appendix

Bart Hobijn
Federico Ravenna
Andrea Tambalotti

Contents

This appendix contains the mathematical details underlying some of the results in the paper. The results are explained in the order in which they appear in the text.

A Section III: Menu Cost Distribution

In the main text, we introduced the menu cost distribution function in the form used by DKW,

$$G_t(\xi) = \begin{cases} 0 & \text{for} \quad \xi < 0 \\ \gamma_{1t} + \gamma_{2t} \tan(\gamma_{3t}\xi + \gamma_{4t}) & \text{for} \quad 0 \leq \xi < \overline{\xi}_t \\ 1 & \text{for} \quad \overline{\xi}_t \leq \xi. \end{cases} \tag{1}$$

To see how this distribution nests the menu cost distribution functions underlying the Calvo and Calvo-Taylor hybrid models, our models (i) and (ii), it is convenient to reparametrize G_t in terms of $\overline{\xi}_t$, $\overline{\phi}$, and $\underline{\phi}$ as

$$\gamma_{1t} = -\gamma_{2t} \tan\left(\left(\underline{\phi} - 0.5\right)\pi\right) \tag{2}$$

$$\gamma_{2t} = 1/\left[\tan\left(\left(\overline{\phi} - 0.5\right)\pi\right) + \tan\left(\left(\underline{\phi} - 0.5\right)\pi\right)\right] \tag{3}$$

$$\gamma_{3t} = \left(\overline{\phi} - \underline{\phi}\right)\pi/\overline{\xi}_t \tag{4}$$

$$\gamma_{4t} = \left(\underline{\phi} - 0.5\right)\pi, \tag{5}$$

where $0 < \overline{\phi} < \underline{\phi} < 1$.

The intuition for this parameterization is most easily explained by referring to figure I. The DKW distribution function is a transformation of the tangent function on $\left[\left(\underline{\phi} - 0.5\right)\pi, \left(\overline{\phi} - 0.5\right)\pi\right]$. This interval is depicted by the arrow denoted by (i) in the figure. The mapping of this interval on the support of the distribution function, i.e. $\left[0, \overline{\xi}_t\right]$,determines the value of the parameters γ_{3t} and γ_{4t}. Arrow (ii) depicts how γ_{1t} is determined. That is, γ_{1t} is such that the value of the distribution function at the minimum of its support equals zero. To assure that the menu cost assumes a value within its support with probability one, γ_{2t} is chosen such that arrow (iii) is of length one.

The menu cost distribution function that underlies both the Calvo model and the Calvo-Taylor hybrid is

$$G_t^*(\xi) = \begin{cases} 0 & \text{for} \quad \xi < 0 \\ \alpha & \text{for} \quad 0 \leq \xi < \overline{\xi}_t \\ 1 & \text{for} \quad \overline{\xi}_t \leq \xi, \end{cases} \tag{6}$$

where for the Calvo model $\overline{\xi}_t$ is infinite at all t, except T when the euro is introduced, and for the Calvo-Taylor hybrid it is finite.

$G_t^*(.)$ is a limiting case of $G_t(.)$ in the sense that $G_t^*(.) \to G_t(.)$ when $\underline{\phi} \to 0$ and $\overline{\phi}$ is chosen according to

$$\overline{\phi} = 0.5 + \frac{1}{\pi} \arctan\left(- \left(\frac{1-\alpha}{\alpha} \right) \tan\left(\left(\underline{\phi} - 0.5 \right) \pi \right) \right). \tag{7}$$

B Section IV: Equilibrium Inflation Dynamics

For the derivation of the equilibrium inflation dynamics and equilibrium price adjustment behavior we write the problem in terms of variables that are constant along the balanced growth path. For this purpose, we define

$$\pi_{j,t}^S = \frac{\Pi_{j,t}^S}{\left[(1+\pi)(1+g)\right]^t},\ v_{j,t}^S = \frac{V_{j,t}^S}{\left[(1+\pi)(1+g)\right]^t}, \tag{8}$$

$$p_{S,t}^* = \frac{P_{S,t}^*}{(1+\pi)^t},\ \text{and } p_{i,t} = \frac{P_{i,t}}{(1+\pi)^t}, \tag{9}$$

for $s \in \{D, E\}$ and $j = 0, \ldots, \infty$.

Given these definitions, write the detrended profits as

$$\pi_{j,t}^S = \left(\frac{p_{S,t-j}^*}{(1+\pi)^j} - \psi \right) \left(\frac{1}{(1+\pi)^j} \frac{p_{S,t-j}^*}{p_{it}} \right)^{-\varepsilon} \left(\frac{p_{it}}{p} \right)^{-\eta} y, \tag{10}$$

where again $s \in \{D, E\}$ and $j = 0, \ldots, \infty$.

Furthermore, we can write the functional equations for the detrended value function as

$$v_{0,t}^D = \max_{p_{D,t}^*} \left\{ \pi_{0,t}^D + \lambda E_t \max\left\{ v_{0,t+1}^D - w\xi, v_{0,t+1}^E - w\xi - wc, v_{1,t+1}^D \right\} \right\} \tag{11}$$

$$v_{0,t}^E = \max_{p_{E,t}^*} \left\{ \pi_{0,t}^E + \lambda E_t \max\left\{ v_{0,t+1}^E - w\xi, v_{1,t+1}^E \right\} \right\} \tag{12}$$

$$v_{j,t}^D = \left\{ \pi_{j,t}^D + \lambda E_t \max\left\{ v_{0,t+1}^D - w\xi, v_{0,t+1}^E - w\xi - wc, v_{j+1,t+1}^D \right\} \right\} \tag{13}$$

$$v_{j,t}^E = \left\{ \pi_{j,t}^E + \lambda E_t \max\left\{ v_{0,t+1}^E - w\xi, v_{j+1,t+1}^E \right\} \right\}, \tag{14}$$

where $\lambda = (1+g)(1+\pi)/(1+r)$.

Since these value functions are defined in terms of variables that are constant along the economy's balanced growth path, we will use this representation of the value functions to solve for the transitional path of the price level in sector i following the announcement of the conversion to the euro.

To solve for the equilibrium inflation dynamics, we need to define the proper state space. The structure of the state space in this model is very similar to that in DKW. The main difference is that it is not only defined as the discrete distribution of firms over the length over which they have not adjusted their prices, but also over the denomination in which they charge their prices.

Let $\theta_{j,t}^{S}$ for $S \in \{D, E\}$ denote the fraction of firms at the *start* of period t that changed their price j periods ago and that charge their price in denomination S. Furthermore, let $\alpha_{j,t}^{S'}$ for $S' \in \{D, E\}$ denote the fraction of firms charging a price set j periods ago in denomination S', which change their price at time t and keep the same denomination. Let $\alpha_{j,t}^{C}$ denote the fraction of firms charging a price set j periods ago in their old domestic currency, which change their price at time t and switch to the euro. Finally, let $\omega_{j,t}^{S}$ for $S \in \{D, E\}$ denote the fraction of firms at the *end* of period t that changed their price j periods ago and that charge their price in denomination S. Here, the end of period refers to the part of the period after which firms have made their pricing decisions. This is the part of the period in which revenue is generated and prices are measured.

The dynamic transition equations for the state are given by the following identities

$$\omega_{0,t}^{E} = \sum_{j=1}^{\infty} \left(\alpha_{j,t}^{E}\theta_{j,t}^{E} + \alpha_{j,t}^{C}\theta_{j,t}^{D}\right) \tag{15}$$

$$\omega_{0,t}^{D} = \sum_{j=1}^{\infty} \alpha_{j,t}^{D}\theta_{j,t}^{D} \tag{16}$$

$$\omega_{j,t}^{E} = \left(1 - \alpha_{j,t}^{E}\right)\theta_{j,t}^{E} \tag{17}$$

$$\omega_{j,t}^{D} = \left(1 - \alpha_{j,t}^{D} - \alpha_{j,t}^{C}\right)\theta_{j,t}^{D} \tag{18}$$

$$\theta_{j+1,t+1}^{S} = \omega_{j,t}^{S} \text{ for } S \in \{D, E\}, \tag{19}$$

where $\omega_{j,t}^{S} \geq 0$ and $\sum_{s=0}^{\infty} \omega_{j,t}^{S} = 1$, since the state represents a distribution of firms. Furthermore, since they represent transition probabilities, $0 \leq \alpha_{j,t}^{S} \leq 1$ for $S \in \{C, D, E\}$.

This definition of the state allows us to define the price level at the end of the period as a function of the state and the prices set by the firms. That is, we can write the measured price level at each point in time as

$$P_{it} = \left[\sum_{S\in\{D,E\}} \sum_{j=0}^{\infty} \omega_{j,t}^{S} \left(\frac{1}{P_{S,t-j}^{*}}\right)^{\varepsilon-1}\right]^{\frac{1}{1-\varepsilon}}. \tag{20}$$

In terms of detrended prices, this is

$$p_{it} = \left[\sum_{S \in \{D,E\}} \sum_{j=0}^{\infty} \omega_{j,t}^{S} \left(\frac{(1+\pi)^j}{p_{S,t-j}^{*}} \right)^{\varepsilon - 1} \right]^{\frac{1}{1-\varepsilon}} \tag{21}$$

which is constant on the balanced growth path.

Solving for the firms' optimal price setting involves characterizing three decisions: (i) whether or not to adjust the price, (ii) whether or not to switch to the euro (in case they are charging prices in the domestic currency), and (iii) what price to charge if the price is adjusted. We will tackle parts (i) and (ii) first and then solve (iii).

A firm that charges its price in euros in period t and set that price j periods ago will adjust its price whenever the menu cost it draws is smaller than the gain in value from adjusting. Mathematically,

$$\xi \leq \left(v_{0,t}^{E} - v_{j,t}^{E} \right) / \, w. \tag{22}$$

The probability of this event depends on the distribution function of menu costs. In particular,

$$\alpha_{j,t}^{E} = G\left(\left(v_{0,t}^{E} - v_{j,t}^{E} \right) / \, w \right). \tag{23}$$

We will denote the expected menu cost for such a firm, conditional on adjusting its price as

$$\Xi_{j,t}^{E} = \int_{0}^{\left(v_{0,t}^{E} - v_{j,t}^{E}\right)/w} \xi dG(\xi).$$

This price adjustment rule is essentially the same as that in DKW.

This is not the case for the a firm that charges its price in the domestic currency, though. Rather than deciding on whether or not to change its price, such a firm decides on whether to change its price and continue to charge it in the domestic currency, change its price and start charging it in euros, or not change its price at all.

If the firm decides to change its price, it will start charging it in euros whenever the value of charging it in euros net of the euro conversion adjustment cost is higher than the value of continuing to charge it in the domestic currency. That is, if the firm adjusts its price, it will convert to the euro whenever

$$v_{0,t}^{E} - cw \geq v_{0,t}^{D}. \tag{24}$$

If this inequality holds strictly, either all firms that charge their prices in domestic currency and adjust their prices will change to euros or they will all keep on charging their prices in the domestic currency. Hence, in that case $\alpha_{j,t}^{D}\alpha_{j,t}^{C}=0$.

A firm that set its domestic currency denominated price j periods ago will adjust its price whenever the menu cost it draws satisfies

$$\xi \leq \max\left\{v_{0,t}^{E}-cw-v_{j,t}^{D}, v_{0,t}^{D}-v_{j,t}^{D}\right\}\big/ w. \tag{25}$$

This allows us to solve for the adjustment probabilities

$$\alpha_{j,t}^{D}=\begin{cases} G\left(\left(v_{0,t}^{D}-v_{j,t}^{D}\right)\big/ w\right) & \text{whenever} \quad v_{0,t}^{D}>v_{0,t}^{E}-cw \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

and

$$\alpha_{j,t}^{C}=\begin{cases} G\left(\left(v_{0,t}^{E}-cw-v_{j,t}^{D}\right)\big/ w\right) & \text{whenever} \quad v_{0,t}^{E}-cw>v_{0,t}^{D} \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

In case of indifference, we assume that firms will switch to the euro.

The expected menu cost, conditional on adjusting the price and still charging it in the domestic currency is

$$\Xi_{j,t}^{D}=\begin{cases} \int_{0}^{\left(v_{0,t}^{D}-v_{j,t}^{D}\right)/w}\xi dG\left(\xi\right) & \text{whenever} \quad v_{0,t}^{D}>v_{0,t}^{E}-cw \\ 0 & \text{otherwise,} \end{cases} \tag{28}$$

while the expected menu cost, conditional on adjusting the price and switching to the euro equals

$$\Xi_{j,t}^{C}=\begin{cases} \int_{0}^{\left(v_{0,t}^{E}-cw-v_{j,t}^{D}\right)/w}\xi dG\left(\xi\right) & \text{whenever} \quad v_{0,t}^{E}-cw>v_{0,t}^{D} \\ 0 & \text{otherwise.} \end{cases} \tag{29}$$

The solution of these adjustment probabilities and expected adjustment costs now allows us to solve for the optimal price $P_{S,t}^{*}$ for $S\in\{D,E\}$. However, we will solve for the optimal detrended price, $p_{S,t}^{*}$, rather than for $P_{S,t}^{*}$. To do so, it is convenient to first rewrite the functional equations that define the value function by substituting in the optimal price adjustment decisions. This yields

$$
\begin{aligned}
(30) \qquad v_{0,t}^{D} &= \max_{p_{D,t}^{*}} \left\{ \pi_{0,t}^{D} + \lambda\alpha_{1,t+1}^{D} v_{0,t+1}^{D} + \lambda\alpha_{1,t+1}^{C} \left(v_{0,t+1}^{E} - wc\right) \right. \\
&\quad \left. + \lambda\left(1 - \alpha_{1,t+1}^{D} - \alpha_{1,t+1}^{C}\right) v_{1,t+1}^{D} - \lambda w \Xi_{1,t+1}^{D} - \lambda w \Xi_{1,t+1}^{C} \right\} \\
(31) \qquad v_{0,t}^{E} &= \max_{p_{E,t}^{*}} \left\{ \pi_{0,t}^{E} + \lambda\alpha_{1,t+1}^{E} v_{0,t+1}^{E} + \lambda\left(1 - \alpha_{1,t+1}^{E}\right) v_{1,t+1}^{E} - \lambda w \Xi_{1,t+1}^{E} \right\} \\
(32) \qquad v_{j,t}^{D} &= \left\{ \pi_{j,t}^{D} + \lambda\alpha_{j+1,t+1}^{D} v_{0,t+1}^{D} + \lambda\alpha_{j+1,t+1}^{C} \left(v_{0,t+1}^{E} - wc\right) \right. \\
&\quad \left. + \lambda\left(1 - \alpha_{j+1,t+1}^{D} - \alpha_{j+1,t+1}^{C}\right) v_{j+1,t+1}^{D} - \lambda w \Xi_{j+1,t+1}^{D} - \lambda w \Xi_{j+1,t+1}^{C} \right\} \\
(33) \qquad v_{j,t}^{E} &= \left\{ \pi_{j,t}^{E} + \lambda\alpha_{j+1,t+1}^{E} v_{0,t+1}^{E} + \lambda\left(1 - \alpha_{j+1,t+1}^{E}\right) v_{j+1,t+1}^{E} - \lambda w \Xi_{j+1,t+1}^{E} \right\},
\end{aligned}
$$

which allows us to derive the first order necessary conditions for the optimal prices $p_{D,t}^{*}$ and $p_{E,t}^{*}$.

The optimal choice of $p_{D,t}^{*}$ implies

$$
\begin{aligned}
(34) \qquad 0 &= \frac{\partial}{\partial p_{D,t}^{*}} \left\{ \pi_{0,t}^{D} + \lambda\alpha_{1,t+1}^{D} v_{0,t+1}^{D} + \lambda\alpha_{1,t+1}^{C} \left(v_{0,t+1}^{E} - wc\right) \right. \\
&\quad \left. + \lambda\left(1 - \alpha_{1,t+1}^{D} - \alpha_{1,t+1}^{C}\right) v_{1,t+1}^{D} - \lambda w \Xi_{1,t+1}^{C} - \lambda w \Xi_{1,t+1}^{D} \right\} \\
(35) \qquad &= \frac{\partial \pi_{0,t}^{D}}{\partial p_{D,t}^{*}} + \lambda \frac{\partial \alpha_{1,t+1}^{D}}{\partial p_{D,t}^{*}} \left(v_{0,t+1}^{D} - v_{1,t+1}^{D}\right) - \lambda w \frac{\partial \Xi_{1,t+1}^{D}}{\partial p_{D,t}^{*}} \\
&\quad \lambda \frac{\partial \alpha_{1,t+1}^{C}}{\partial p_{D,t}^{*}} \left(v_{0,t+1}^{E} - wc - v_{1,t+1}^{D}\right) - \lambda w \frac{\partial \Xi_{1,t+1}^{C}}{\partial p_{D,t}^{*}} \\
(36) \qquad &\quad + \lambda\left(1 - \alpha_{1,t+1}^{D} - \alpha_{1,t+1}^{C}\right) \frac{\partial v_{1,t+1}^{D}}{\partial p_{D,t}^{*}}.
\end{aligned}
$$

Since the envelope theorem implies

$$
\begin{aligned}
(37) \qquad 0 &= \lambda \frac{\partial \alpha_{1,t+1}^{D}}{\partial p_{D,t}^{*}} \left(v_{0,t+1}^{D} - v_{1,t+1}^{D}\right) - \lambda w \frac{\partial \Xi_{1,t+1}^{D}}{\partial p_{D,t}^{*}} \\
(38) \qquad &= \lambda \frac{\partial \alpha_{1,t+1}^{C}}{\partial p_{D,t}^{*}} \left(v_{0,t+1}^{E} - wc - v_{1,t+1}^{D}\right) - \lambda w \frac{\partial \Xi_{1,t+1}^{C}}{\partial p_{D,t}^{*}},
\end{aligned}
$$

the first order condition simplifies to

$$
(39) \qquad 0 = \frac{\partial \pi_{0,t}^{D}}{\partial p_{D,t}^{*}} + \lambda\left(1 - \alpha_{1,t+1}^{D} - \alpha_{1,t+1}^{C}\right) \frac{\partial v_{1,t+1}^{D}}{\partial p_{D,t}^{*}}.
$$

The partial $\frac{\partial v_{j,t+1}^{D}}{\partial p_{D,t}^{*}}$ can be derived in a similar way. It is

$$
(40) \qquad \frac{\partial v_{j,t+1}^{D}}{\partial p_{D,t}^{*}} = \frac{\partial \pi_{j,t}^{D}}{\partial p_{D,t}^{*}} + \lambda\left(1 - \alpha_{j+1,t+1}^{D} - \alpha_{j+1,t+1}^{C}\right) \frac{\partial v_{j+1,t+1}^{D}}{\partial p_{D,t}^{*}}.
$$

Solving the optimality condition through forward recursion yields

$$0=\frac{\partial \pi^{D}_{0,t}}{\partial p^{*}_{D,t}}+\sum_{j=1}^{\infty}\lambda^{j}\prod_{s=1}^{j}\left(1-\alpha^{D}_{j+s,t+s}-\alpha^{C}_{j+s,t+s}\right)\frac{\partial \pi^{D}_{j,t+j}}{\partial p^{*}_{D,t}} \tag{41}$$

and since

$$\frac{\partial \pi^{D}_{j,t+j}}{\partial p^{*}_{D,t}}=\left[\frac{1-\varepsilon}{(1+\pi)^{j}}+\varepsilon\frac{\psi}{p^{*}_{D,t}}\right]\left(\frac{1}{(1+\pi)^{j}}\frac{p^{*}_{D,t}}{p_{it+j}}\right)^{-\varepsilon}\left(\frac{p_{it+j}}{p}\right)^{-\eta}y, \tag{42}$$

the first order condition implies

$$p^{*}_{D,t}=\frac{\varepsilon}{\varepsilon-1}\psi\frac{\sum_{j=0}^{\infty}\chi^{D}_{j,t}\,(1+\pi)^{j}}{\sum_{j=0}^{\infty}\chi^{D}_{j,t}}, \tag{43}$$

where

$$\chi^{D}_{j,t}=\begin{cases}1 & \text{for}\quad j=0\\ \lambda^{j}\prod_{s=1}^{j}\left(1-\alpha^{D}_{s,t+s}-\alpha^{C}_{s,t+s}\right)(1+\pi)^{\varepsilon j}\,p^{\varepsilon-\eta}_{i,t+j} & \text{for}\quad j>0.\end{cases} \tag{44}$$

Similarly, we can solve for the optimal price charged in euros

$$p^{*}_{E,t}=\frac{\varepsilon}{\varepsilon-1}\psi\frac{\sum_{j=0}^{\infty}\chi^{E}_{j,t}\,(1+\pi)^{j}}{\sum_{j=0}^{\infty}\chi^{E}_{j,t}}, \tag{45}$$

where

$$\chi^{D}_{j,t}=\begin{cases}1 & \text{for}\quad j=0\\ \lambda^{j}\prod_{s=1}^{j}\left(1-\alpha^{E}_{s,t+s}\right)(1+\pi)^{\varepsilon j}\,p^{\varepsilon-\eta}_{i,t+j} & \text{for}\quad j>0.\end{cases} \tag{46}$$

In terms of the non-transformed prices, we thus obtain

$$P^{*}_{S,t}=\frac{\varepsilon}{\varepsilon-1}\sum_{j=0}^{\infty}\Omega^{S}_{j,t}\Psi_{t+j}\text{ where }\Omega^{S}_{j,t}=\frac{\chi^{S}_{j,t}}{\sum_{q=0}^{\infty}\chi^{S}_{q,t}}, \tag{47}$$

for $S\in\{D,E\}$. This is the result used in the main text.

C Section V: Calibration of Price Adjustment Frequencies

Because our calibration pertains to the steady state in which firms do not switch the denomination of their prices, we ignore the currency denomination dimension in this derivation. Our goal is to derive an expression for the probability of adjusting the price x times in 12 monhs.

Let $q_{x,y,j}$ denote the probability that a firm adjusts its price x times over the next y periods conditional on not having adjusted its price for j periods. Let α_j denote the probability of adjusting the price when a firm has not adjusted its price for j periods.

These probabilities can be derived using a version of the Chapman-Kolmogorov equation for discrete time and discrete state Markov processes. The particular application here yields that

$$q_{x,y,j} = (1 - \alpha_j)\, q_{x,y-1,j+1} + \alpha_j q_{x-1,y-1,0} \tag{48}$$

The intuition for this result is that there are two ways to adjust x times from now during the next y periods. The first, which happens with probability $(1 - \alpha_j)$, is that a firm does not adjust its price in the current period and will thus have to adjust its price x times in the remaining $y - 1$ periods. The second, which happens with probability α_j, is that the firm adjusts its price in the current period and has $y - 1$ periods left to adjust its price another $x - 1$ times.

Since the model implies that a firm can at maximimum adjust its price once per period, $q_{x,y,j} = 0$ for $x > y$ and all j. Note also that $q_{0,0,j} = 1$ for all j. These latter two results can be used to initialize the recursion implied by the Chapman-Kolmogorov equation above.

If a period is a month, then the probability that a firm that hasn't adjusted its price for j months adjusts its price x times in the subsequent year is given by $q_{x,12,j}$. Let ω_j be the steady state fraction of firms that have not adjusted their prices for j months. Then, the fraction of firms that will adjust their price x times in a year in steady state, which we denote by $Q_{x,12}$, equals

$$Q_{x,12} = \sum_{j=0}^{\infty} \omega_j q_{x,12,j}. \tag{49}$$

This is what we use for our calibration. In particular, the data for the Dutch restaurant sector we consider reports the empirical equivalents of $Q_{0,12}$, $Q_{1,12}$, and $Q_{2,12} + Q_{3,12} + Q_{4,12}$.

D Section V: Numerical Solution Method

For the numerical solution of our model we use an "extended path" method. This method has been applied in other studies of transitional dynamics, like Greenwood and Yorukoglu (1997). We assume that the economy starts off in period 0 in the steady state in which everyone charges prices in the domestic currency and charging prices in euros is not an option. In period 0, the conversion to the euro at time T is announced. We will solve for the transitional path of the economy under the assumption that at time $\overline{T} > T > 0$ the sector has converged to its new steady state. This new steady state is the one in which all firms charge their prices in euros.

The numerical solution method works as follows

1. Start with a guess for the equilibrium price path $\{p_{i,t}\}_{t=0}^{\overline{T}}$.

2. Solve the optimal price setting response for the firms. This is done using the value function iterations, (30) through (33), the optimal price setting rules, (43) and (45), and the transition equations for the state space, (15) through (19) and (23), (26) and (27).

3. Use the new path of prices and the state space to solve the price level identity, (21) and obtain a new equilibrium price path $\left\{p'_{i,t}\right\}_{t=0}^{\overline{T}}$.

4. Repeat steps 2 and 3 until $\{p_{i,t}\}_{t=0}^{\overline{T}} \to \left\{p'_{i,t}\right\}_{t=0}^{\overline{T}}$.

GAUSS code for the implementation of this numerical procedure is available upon request.

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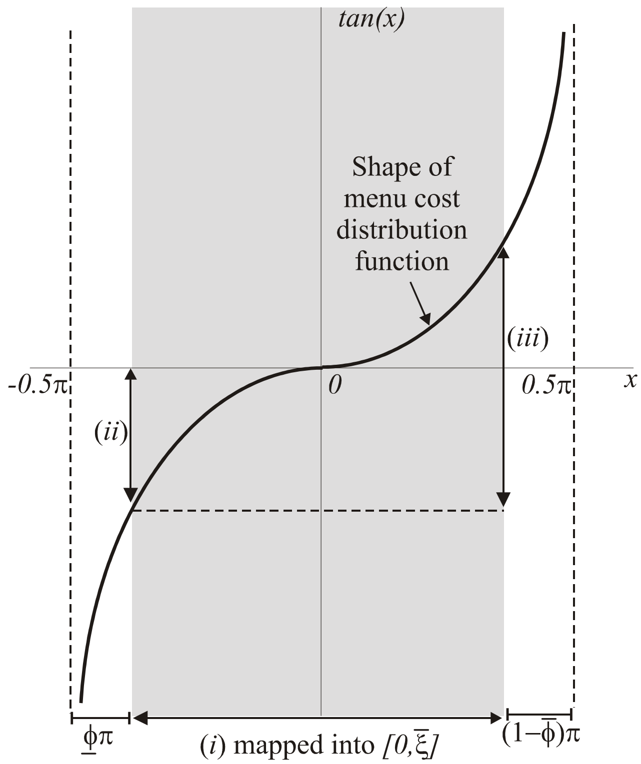

Figure I: DKW menu cost distribution